Photon Localization and Vacuum Noise in Optical Measurements

Alexander S. Shumovsky
Physics Department, Bilkent University, Bilkent, Ankara, 06533, Turkey

Description of detection and emission in terms of the photon localization is discussed. It is shown that the standard representation of the plane waves of photons should be revised to take into consideration the boundary conditions caused by the presence of quantum emitters and detectors. In turn, the change of the boundary conditions causes spatially inhomogeneous structure of the electromagnetic vacuum which leads to the increase of the vacuum noise over the level predicted within the framework of the model of plane waves of photons.

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Since the early days of quantum theory of light, the problem of localizing photons has attracted a great deal of interest (e.g., see [1,2] for recent discussion). The point is that the photon operators of creation and destruction are defined in all space. At the same time, the intensity measurement by means of a photodetector with finite sensitive area σ presupposes a kind of the photon localization, at least in vicinity of σ [3]. The transformation of photons into an electronic signal in photodetectors is not the only onion in the stew. Another example of some considerable interest is provided by the emission and absorption of radiation by atoms and molecules.

We now note that the electromagnetic field is usually quantized as though it is free and propagates in empty space. This model leads to well-known plane waves of photons, corresponding to the quantized translation invariant solutions of the homogeneous Helmholtz wave equation [1,2,4]. The presence of atoms or surfaces which are able to interact with photons leads to the change of boundary conditions and hence to the violation of translational symmetry. For example, the presence of a single point-like atom causes the multipole structure of the field which can be described in terms of quantized spherical waves [5,6]. The latter case is specified by the $SO(3)$ symmetry rather than translational symmetry.

In this note we show that the taking account of the boundary conditions at both emission and detection of the field from the plane waves of photons lead to more adequate picture of the photon localization. We also show that the change of the boundary conditions strongly influences the zero-point oscillations of the field strengths which causes a deterioration of quantum limit of precision of measurements.

Consider first an atom located at the origin of the reference frame spanned by the complex base vectors

$$\vec{\chi}_{\pm} = \mp \frac{\vec{e}_x \pm i\vec{e}_y}{\sqrt{2}}, \qquad \vec{\chi}_0 = \vec{e}_z.$$

These vectors formally coincide with the states of spin 1 of the photon [7]. Since the quantum electrodynamics defines the spin states of photons as the polarization [8], we can choose to interpret $\vec{\chi}_{\pm}$ as the unit vectors of transversal polarization with either positive or negative helicity, while $\vec{\chi}_0$ is the unit vector of linear polarization in the z-direction. The third spin state is forbidden in the case of plane waves of photons due to the translational invariance, while allowed in the case of spherical waves of photons [5,6]. An arbitrary vector $\vec{A}$ is expanded in this basis as follows

$$\vec{A} = \sum_{\mu} (-1)^{\mu} \vec{\chi}_{-\mu} A_{\mu}.$$

The positive-frequency part of the electric field strength of the monochromatic multipole field is then defined as having components [5,8]

$$E_{\mu}(\vec{r}) = ik\gamma \sum_{\lambda} \sum_{j=1}^{\infty} \sum_{m=-j}^{j} V_{\lambda jm\mu}(\vec{r}) a_{\lambda jm}, \qquad (1)$$

where $\lambda = E, M$ denotes the type of radiation (either electric or magnetic), γ is the normalization factor, j, m are the angular momentum quantum numbers. In the classical picture, the complex field amplitudes are defined by the source [4]. To obtain the quantum counterpart, we have to subject these amplitudes to the Weyl-Heisenberg commutation relations [5]

$$[a_{\lambda jm}, a^{+}_{\lambda' j' m'}] = \delta_{\lambda\lambda'} \delta_{jj'} \delta_{mm'}. \qquad (2)$$

The mode functions in (1) have the form [5,8]

$$\begin{aligned} V_{Ejm\mu}(\vec{r}) &= \frac{1}{\sqrt{2j+1}} \\ &\times [\sqrt{j} f_{j+1} \langle 1, j+1, \mu, m-\mu | jm \rangle Y_{j+1, m-\mu} \\ &- \sqrt{j+1} f_{j-1} \langle 1, j-1, \mu, m-\mu | jm \rangle Y_{j-1, m-\mu}], \\ V_{Mjm\mu}(\vec{r}) &= f_j(kr) \langle 1, j, \mu, m-\mu | jm \rangle Y_{j, m-\mu}, \end{aligned} \qquad (3)$$

where the radial function $f_\ell(kr)$ is represented either by the spherical Bessel function $j_\ell(kr)$, in the case of standing waves in a spherical cavity, or by the spherical Hankel functions of the first and the second kind, describing the outgoing and converging spherical waves respectively. Here $\langle \cdots | jm \rangle$ denotes the Clebsch-Gordon coefficient of vector addition of the spin and orbital parts of the total angular momentum and $Y_{\ell, m-\mu}$ is the spherical harmonics.

In view of (2), the zero-point oscillations of the electric field strength (1) have the form

$$\begin{aligned} C_E(\vec{r}) &\equiv \langle 0 | (\vec{E}(\vec{r}) + \vec{E}^{+}(\vec{r}))^2 | 0 \rangle \\ &= \sum_{\mu} \langle 0 | E_{\mu}(\vec{r}) E^{+}_{\mu}(\vec{r}) | 0 \rangle \\ &= (k\gamma)^2 \sum_{\mu} \sum_{\lambda, j, m} |V_{\lambda jm\mu}(\vec{r})|^2. \end{aligned} \qquad (4)$$

To make a comparison, we remind here that the zero-point oscillations in the case of the monochromatic plane waves have the form [1,2]

$$C_{plane} = 2(k\gamma')^2 \qquad (5)$$

everywhere.

It is seen that, unlike (5), the spherical waves of photons have the spatially inhomogeneous zero-point oscillations. It is now a straightforward matter to show that (4) strongly exceeds the standard level, given by (5), at least in some vicinity of the origin (atom), while tends to (5) as $kr \gg 1$. A more detailed examination shows that $C_E(\vec{r}) \gg C_{plane}$ at $kr \leq 2.5$ which gives the distance of the order of 0.3Λ where Λ is the wavelength. Let us stress that this distance is of the order of typical interatomic

separation in experiments with trapped Ridberg atoms [9].

We now stress that the above results have been obtained under the only assumption that the atom exists at the origin independent of whether we use it for emission or detection. Therefore, the strong increase of the vacuum noise in vicinity of the atom should influence both the emission and detection processes. As a simple model of complete Hertz-type optical experiment, we consider the two identical atoms separated by distance d. The first atom (source) is prepared initially in the excited state of some multipole transition, while the second atom (detector) is in the ground state. Then, the measurement consists in the emission and successive absorption of a photon.

It is clear that in order to take into account the initial localization of photon within the source, the radiation should be described in terms of the outgoing spherical wave focused on the first atom. In turn, the final localization within the detector, assumes the converging spherical wave focused on the second atom. To combine these two processes into the common picture, we have to describe the filed as the superposition of outgoing and converging waves. The coefficients of this superposed state should be defined by the boundary conditions for the real radiation field. Taking into account the recent investigation [10], we anticipate that this model obeys the causality principle of the electrodynamics. In view of the position dependence in (3) and (4), it is clear that, at far distances ($d \gg \Lambda$), the major contribution into the vacuum noise of measurement comes from the detecting atom, while, at intermediate and short distances, the noise of measurement is increased due to the influence of the source atom.

Consider now the measurement of a plane photon by a photodetector. At far distances, the photon is described by a unique wave vector $\vec{k}$. The Mandel's localization in vicinity of the sensitive area σ [3] assumes that the wave converges to σ. This means that there is a variety of directions of the wave vectors near σ, although all of them have the same length. This picture, based on the taking account of the boundary conditions, can be described by a proper expansion over spherical waves. In view of the above discussion, it should lead to the increase of the vacuum noise of measurement over the level (5).

Let us briefly summarize the results. First of all, it is clear that the above results represent an extension and detailing of the Mandel's model of the photon localization [3]. It has been shown that the description of the photon localization in the process of detection and emission needs more adequate consideration of the boundary conditions, leading to a violation of the translation invariance inherent in the conventional model of the plane waves of photons. This violation leads to the qualitative change of the structure of the electromagnetic vacuum state. In particular, the zero-point oscillations are concentrated in vicinity of atoms, molecules, photodetectors and other local objects which are able to interact with photons. The level of the zero-point oscillations in vicinity of the emitting and measuring devices can strongly exceeds that calculated as though the field consists of the plane waves of photons. This leads to a deterioration in the estimation of the quantum limit of precision of measurement.

The above results can be important for different quantum optical measurements especially in the engineered entanglement based on the trapped atoms [9], in the experiments with atomic beams and single-atom lasers [11] and in the quantum polarization measurement [12].

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